

B.Sc Kart - 1

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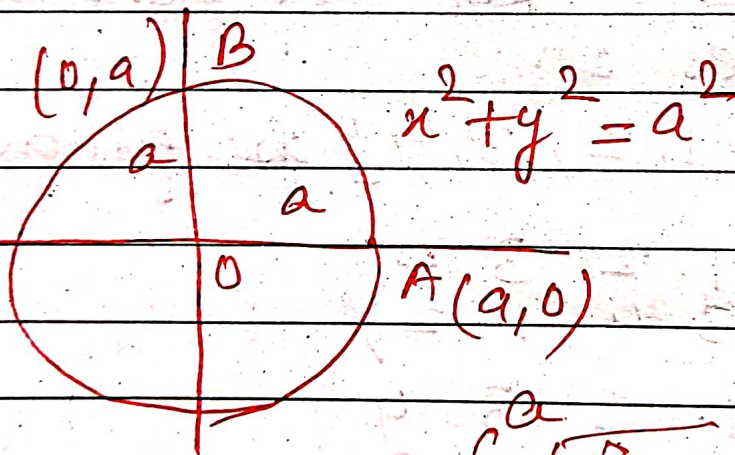
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Q1. Find the perimeter of that circle

$$x^2 + y^2 = a^2$$

Solⁿ The curve is symmetrical about both axes. Hence the perimeter is four times the length of arc BA.

Now corresponding to the point B, the abscissa is zero and corresponding to A, the abscissa is a.



$$\therefore S = 4 \int_0^a \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

Now differentiating $x^2 + y^2 = a^2$, we get

$$2x + 2y \frac{dy}{dx} = 0 \quad \text{OR} \quad \frac{dy}{dx} = -\frac{x}{y}$$

$$\begin{aligned} \therefore 1 + \left(\frac{dy}{dx}\right)^2 &= 1 + \frac{x^2}{y^2} = \frac{x^2 + y^2}{y^2} \\ &= \frac{a^2}{y^2} \end{aligned}$$

$$\begin{aligned} \therefore S &= 4 \int_0^a \frac{a}{y} dx = 4a \int_0^a \frac{dx}{\sqrt{a^2 - x^2}} \\ &= 4a \left[\sin^{-1} \frac{x}{a} \right]_0^a = 4a \frac{\pi}{2} = 2\pi a \end{aligned}$$